$e^{ix} = \cos x + i \sin x$

Euler's discoveries

Euler worked in almost all areas of mathematics: geometry, calculus, trigonometry, algebra, and number theory, not to mention continuum physics, lunar theory, etc. His importance in the history of mathematics cannot be overstated: his works correspond to about 80 quarto volumes.

Mathematical notation

Euler introduced and popularized several notational conventions, including the concept of a function: f(x), cos(x), sin(x), etc. He introduced the letters e as the base of the natural logarithm (Euler's number), Σ for sums and i ($\sqrt{-1}$). He popularized π (3.1415...).

Analysis

With the support of his friends Bernoullis, Euler focused on calculus, at the forefront of 18th century, and made key contributions. He frequently used the logarithm function as a tool and expressed logarithmic functions in terms of power series.

He is also well known for his frequent use and development of power series i.e. functions as infinite sums of powers of the variable, such as $e^x = \sum_{n=0} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
As an other example, he used $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{5!} + \cdots$ to solve for the first time the famous Basel Problem: $\sum_{n=1} \frac{1}{n^2} = \lim \frac{1}{1^2} \cdot \frac{1}{2^2} \cdots \frac{1}{n^2} = \ldots \pi^2/6$.

Euler introduced the use of the exponential function and logarithms in analytic proofs. He defined the exponential function for complex numbers and discovered its relation to the trigonometric functions. For any real number x, Euler's formula is $e^{ix} = \cos x + i \sin x$ which implies "the most remarkable formula in mathematics" (R. Feynman), Euler's identity:

$$e^{i\pi} + 1 = 0$$

In addition, Euler elaborated the theory of higher transcendental functions by introducing the gamma function.

Number theory

Euler pioneered the use of analytic methods to solve number problems and his work is extremely useful today in cryptography. For example, he studied the nature of prime distribution (1,2, 3, 5, 7, 11, ...) with ideas in analysis.

Euler proved Fermat's theorems, and made distinct contributions to Lagrange's four-square theorem. He invented the totient function which assigns to a positive integer n the number of positive integers less than n and coprime to n. He discovered Euler's theorem :

$$a^{\varphi(n)} \equiv 1 \pmod{n}$$

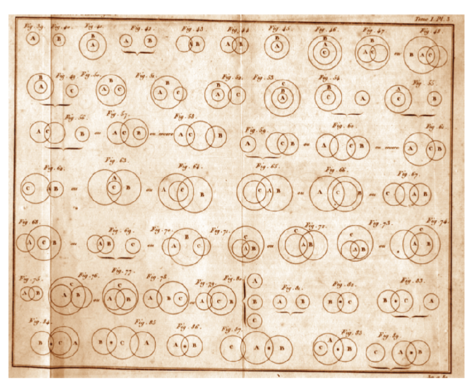
Graph theory, topology, combinatorics, operations research

In 1736 Euler solved the 7 bridges of Königsberg's problem. Königsberg has a river with two islands and seven bridges. Euler explained why it is not possible to walk on each bridge exactly once, and return to the starting point. His solution is considered the birth of graph theory and therefore of operations research.

He introduced the "Euler characteristic" notion of a space by finding a formula relating the number of edges, vertices, and faces of polyhedron. He proved Newton's identities, with an impact on Galois theory, group theory, combinatorics, general relativity, etc.

Applied mathematics, numerical analysis

Some of Euler's greatest successes were in using analytical methods to solve real world problems. He integrated Leibniz's differential calculus with Newton's method of fluxions, and developed tools to apply calculus to physical problems. He improved the numerical approximation of integrals, inventing the Euler approximations, Euler's method and the Euler-Maclaurin formula. He also facilitated the use of differential equations.



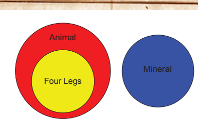
Physics and astronomy

Euler developed the Euler-Bernoulli beam equation, which became a cornerstone of engineering. He successfully applied his analytical tools to problems in classical mechanics and celestial problems, e.g. determining the orbits of comets, calculating the parallax of the sun, or determining accurate longitude tables.

In addition, Euler made important contribution to optics. He disagreed with Newton's corpuscular theory of light, which was then the prevailing theory. His papers on optics helped ensure that the wave theory of light became the dominant mode of thought, at least until the development of the quantum theory of light.

Logic

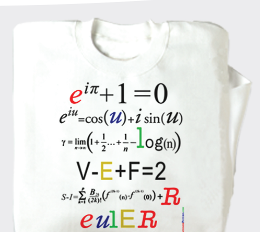
He is also credited with using closed curves to illustrate syllogistic reasoning. These diagrams are now known as Euler diagrams, and do not need to show all possible intersections.

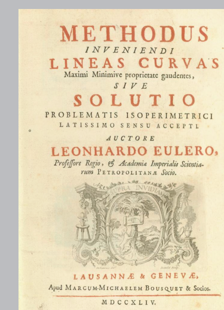


Euler diagrams

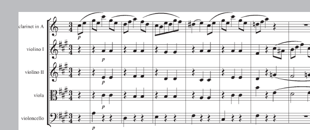
What is named after Euler

(φ) function Eulerian path Euler's method AMS Euler typeface Euler's idoneal numbers
Euler's number e ≈ 2.71828 Euler-Mascheroni constant Euler-Bernoulli beam equation
Euler Medal Euler-Maclaurin formula Euler approximation Euler's rule Euler
Euler's formula v − e + f = 2 Euler's line Euler angles Euler-Rodrigues parameters Euler's
Euler's constant γ ≈ 0.577216 The Euler integrals Euler squares Euler
Euler diagram Euler derivative Euler's conjecture Euler's formula χ (S²) = F − E + V = 2
Euler's equations Euler equations in fluid dynamics Euler phi (φ) function The
Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ Euler's rigid body equations
2002 Euler asteroid Euler-Tricomi equation Euler
Euler's idoneal numbers Euler's identity Euler brick Euler
Euler spline Euler product Euler
Euler's totient function





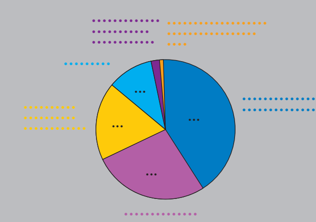
Euler's Methodus inveniendi lineas curvas - the first systematic treatise on the calculus of variations.



One of Euler's morer unusual interests was the application of mathematical ideas to music.

Technical range

Euler's writings break down approximately as follows:



From Euler to art?

Euler's characteristic is also key in tessellation (shapes that fills the plane), together with Swiss mathematician Schläfli's symbol, to explain why there are only 17 possible "shapes" of wallpapers, or ties:

$e^{i\pi} + 1 = 0$

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