

# Euler and fluids

## Fluid dynamics

Fluid dynamics study fluids -liquids and gases- in motion. It has a wide range of applications, including forces and moments on aircraft, the mass flow rate of petroleum through pipelines, weather patterns, ...

## Euler equations in fluid dynamics

In fluid dynamics, the Euler equations govern the motion of a compressible inviscid fluid (liquids and gases). They basically say that mass, momentum and energy are conserved. The famous Bernoulli's principle (Daniel Bernoulli) states that an increase in velocity occurs simultaneously with decrease in pressure and it can be derived from Euler's equations. Euler equations are necessary to simulate fluid and optimise shapes of cars, planes, boats.

In differential form, the equations are:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u) u + \nabla p = 0 \quad \frac{\partial E}{\partial t} + \nabla \cdot (u(E + p)) = 0 \quad \frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = 0$$

where  $E = \rho e + \rho(u^2 + v^2 + w^2) / 2$  is the total energy per unit volume ( $e$  is the internal energy per unit mass for the fluid),  $p$  is the pressure,  $u$  the fluid velocity and  $\rho$  the fluid density. The second equation includes the divergence of a dyadic tensor, and may be clearer in subscript notation:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0$$

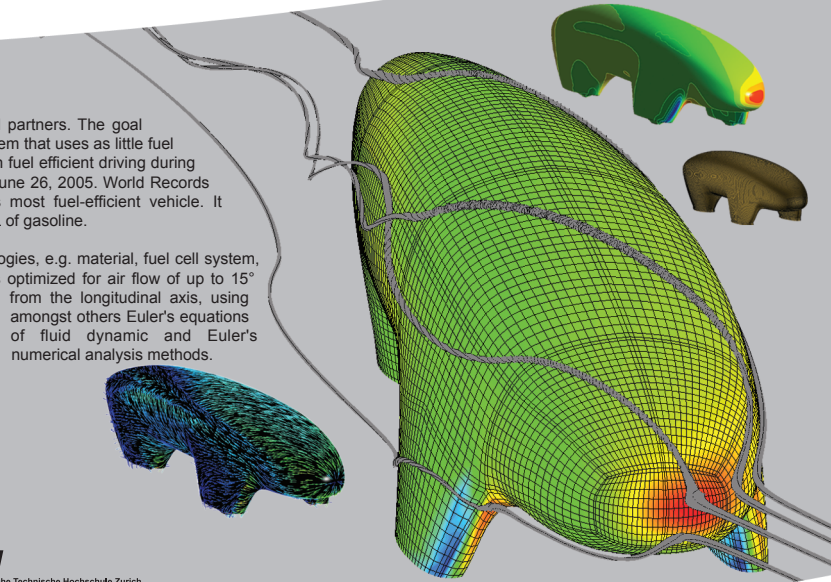
## Relativistic Euler equations

Relativistic Euler equations refer to compressible fluid flow when velocities approach the speed of light.

## PAC Car II

PAC-Car II is a joint project of ETH Zurich and partners. The goal was to build a vehicle powered by a fuel cell system that uses as little fuel as possible. PAC Car II set a new world record in fuel efficient driving during the Shell Eco-marathon in Ladoux (France) on June 26, 2005. World Records (TM) have certified PAC-Car II as the world's most fuel-efficient vehicle. It reached 5385 km with hydrogen equivalent to 1 L of gasoline.

The design of this car used various new technologies, e.g. material, fuel cell system, control system, and fluid dynamics. Its shape is optimized for air flow of up to 15° from the longitudinal axis, using amongst others Euler's equations of fluid dynamic and Euler's numerical analysis methods.



www.paccar.ethz.ch



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## Euler cavitation number

The Euler cavitation number is a dimensionless number used in flow calculations. It expresses the relationship between a flow's pressure and kinetic energy (energy due to its motion). It can for example link the Geneva jet (the highest in the world) pressure, energy and height.

where  
 $\rho$  is the density of the fluid.  
 $p$  is the local pressure.  
 $p_v$  is the vapor pressure of the fluid.  
 $V$  is a characteristic velocity of the flow.

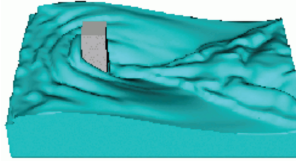
It is defined as

$$Ca = \frac{p - p_v}{\frac{1}{2} \rho V^2}$$



## Traffic

Traffic can be treated as a continuous fluid. Traffic engineers apply the rules of fluid dynamics to traffic flow, likening it to the flow of a fluid in a pipe. Traffic jam simulations have shown that in heavy but free flowing traffic, jams can arise spontaneously, triggered by minor events such as an abrupt steering maneuver by a single motorist. Such a situation is likened to the freezing of supercooled fluid by traffic scientists.



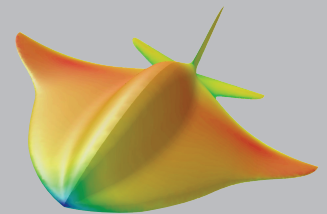
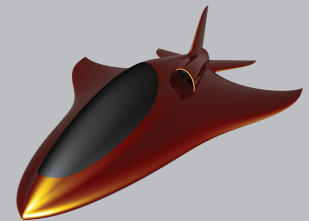
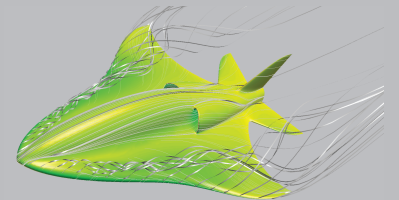
Modern computer simulation of river flow.



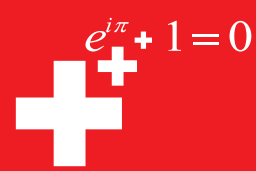
The Euler-Tricomi equation is a linear partial differential equation useful in the study of transonic flow.

## Smartfish

The objective of SmartFish is to develop and commercialise a revolutionary general aviation aircraft technology that is highly innovative in terms of safety, economy and emotion. This technology can be used for a wide range of applications as it can accommodate up to 20 passengers. In order to validate the concept, a two-seater prototype now has to be built. Computational fluid dynamics simulations have been carried out in order to define the optimum shape. The different designs have been built and tested. The excellent flight qualities and performances have been proven.



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