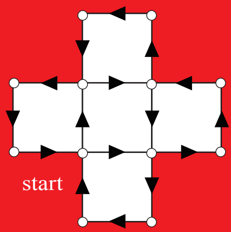


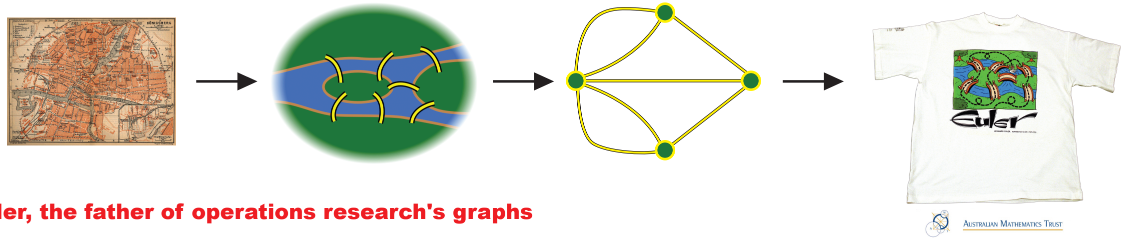
Euler, the father of graph theory and operations research

Euler's solution of the seven bridges of Königsberg problem

The city of Konigsberg (now Kaliningrad, Russia) is set on a river, with 2 islands and 7 bridges. Is it possible to take a journey across all 7 bridges and return to the starting point without crossing any bridge more than once? In 1736, Euler proved that it was not possible.

The key of Euler's approach was to replace each landmass with a dot and each bridge with a line, namely to draw the first "graph" of graph theory.

He realized that the problem could be solved in terms of the degrees of the dots, i.e. the number of lines touching them, and he proved that a circuit of the desired form (called an Eulerian circuit or an Euler tour) is possible if and only if there are no dots of odd degree. Since the graph corresponding to Konigsberg has four dots of odd degree, it cannot have an Eulerian circuit. Traversing all bridges but with different starting and ending point (Eulerian path or Euler walk) was impossible too in Konigsberg because Euler proved that such a path exists if and only if the starting and ending points both have an odd degree or none has one.



Euler, the father of operations research's graphs

His solution was the first theorem of graph theory, combinatorics, and topology, fields that are used in operations research (OR). OR is an interdisciplinary science applying mathematical modeling, statistics, algorithms, etc for decision making in complex real-world problems. Typical problems are:

- Itinerary optimization, for example for cash pick-up, orders delivery, street cleaning or parking fee control: 2 key problems are the shortest path to go to each place (the Salesman problem) and the shortest path to pass by each street (China Postman Problem).
- Optimal programming of green/red lights at junctions.
- Optimal green waves of the red lights in order to accelerate the traffic and minimize the traffic jams
- Optimal scheduling of trains/flights in order to minimize the waiting time in transit places
- Optimal location of mobile phone stations
- Optimal scheduling of schools, universities, projects, manufacturing, airlines staff, etc

The example of the Swiss Federal Railways SBB/CFF scheduling and itinerary optimization software

Switzerland's public transport is the densest in the world, with its trains, buses, boats, funiculars, etc. The Swiss Federal Railways SBB/CFF is equipped with state-of-the-art OR software to optimize scheduling, individual itineraries, and control.

Suppose that you would like to know how to go from Euler-street no 8 in Basel to the famous Zermatt village and ski around the Matterhorn. The SBB/CFF website www.sbb.ch tells you that you first need to walk 11 minutes to the tramway station. When the tramway arrives at the railway station, you walk 6 minutes to the train. The train leaves 2 minutes later. After changing train in Brig in 7 minutes, you arrive in Zermatt.

Details - Connection 2

Station/Stop	Date	Time	Platform	Travel with	Comments
4051 Basel, Eulerstrasse 8 Basel, Schützenhaus	21.01.07			walk	11 min.
Basel, Schützenhaus Basel Bahnhof SBB		dep 06:16 arr 06:19		Tram 1	Low-floor tramway, Direction: Basel Bahnhof SBB
Basel Bahnhof SBB Basel SBB				walk	6 min., Y
Basel SBB Brig		dep 06:27 arr 08:58	10 9	CIS 45	Cisalpino, RE
Brig Brig MGB-fo/bvz				walk	7 min., Y
Brig MGB-fo/bvz Zermatt		dep 09:05 arr 10:24	12	R 225	Regio

Duration: 4:19; runs Sa, Su, public holiday data; also 1. May

» Show intermediate stops » Print view » Fare/Ticket » Ticket+Hotel

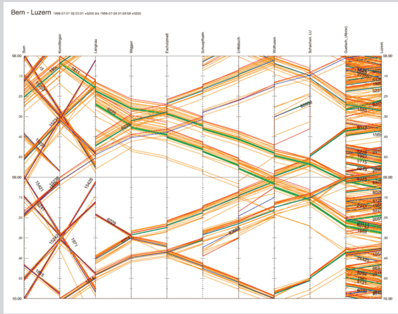
» Top of page

SBB CFF FFS

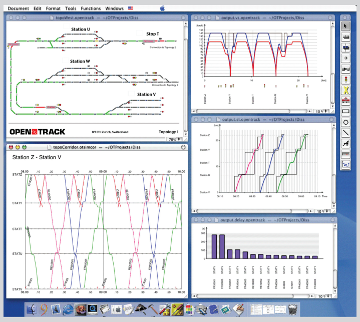
Softwares that optimize scheduling or itineraries use complex Operations Research algorithms. Developed at the Swiss Federal Institute of Technology (ETH) in cooperation with SBB/CFF, the OpenTrack software simulates a railway network; it is used in different countries to determine the requirements for a network's infrastructure, analyze the robustness of timetables, the effect of delays, etc. The software OpenTimeTable focuses on analyzing and optimizing the timetables, creating more attractive services and better utilizing infrastructure.

In OpenTimeTable, the blue lines are the mean effective timetables, the orange lines are the daily train-runnings, the red lines are the scheduled runnings and the green areas indicate the standard deviation of the delays.

OPEN TIMETABLE



OPEN TRACK



OpenTrack produces a number of outputs in text and/or graphic form

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

The "Chinese Postman Problem":

how to find the shortest path passing by all streets and coming back?

This was originally studied by the Chinese mathematician Mei Ko Kuan. Cities rarely have Eulerian circuits therefore an optimal itinerary is required. The streets can be modeled by dots, and the lines correspond to junctions.

Today, such problems are generated by various sources: waste pick-up, street cleaning, parking fee inspection, etc. Many softwares are now available to solve such problems.

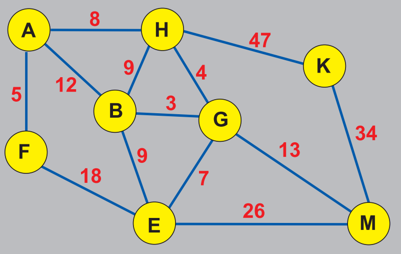
The "Salesman Problem":

how to find the shortest itinerary passing by all shops in all cities and coming back?

Though it somehow appears similar to the Chinese Postman Problem, it is conceptually very different and much more difficult to solve, and not only because the number of possible itineraries is much larger. The cities can be modeled by dots, and the lines correspond to streets. Therefore, the problem is the optimal itinerary passing through all the dots.

Graphs

In the Chinese Postman problem, the dots correspond to streets and the lines to the roads between cities or shops; the distance or the cost is indicated for each line. In the Salesman problem, the dots correspond to cities or shops and the lines correspond to the junctions. The length of each street would then be indicated for each dot.



$$e^{i\pi} + 1 = 0$$

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra
State Secretariat for Education and Research

Switzerland.