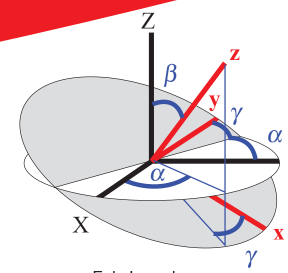
Euler, a physicist



Euler's angles

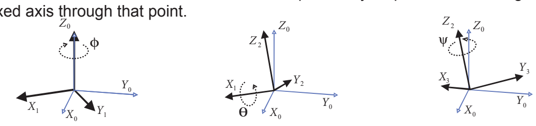
Euler's angles, Euler's rotation theorem and Euler's equations

Euler's angles are the classical way of representing rotations in 3-dimensional Euclidean space.

To be concrete, consider a brick, set on a table, that is then pivoted around one corner. The before and after positions of the brick are characterized by the six edges of the brick meeting the pivot corner.

The Euler angles are three angles between the before and after edges. If, for example, a satellite has spin control in two orthogonal directions, then reorienting the satellite can be accomplished by using the Euler angles directly, in the moving axes definition.

Euler's rotation theorem states that, in 3D space, any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a rotation about a fixed axis through that point.



A rotation represented by Euler angles with (φ,θ,ψ) = (- 60°, 30°, 45°) using the 3-1-3 general (co-moving axes) convention.

Euler's equations describe the rotation of a rigid body in a frame of reference fixed in the rotating body

$$I_1\omega_1 + (I_3 - I_2)\omega_2\omega_3 = N_1 \qquad I_2\omega_2 + (I_1 - I_3)\omega_3\omega_1 = N_2 \qquad I_3\omega_3 + (I_2 - I_1)\omega_1\omega_2 = N_3$$

where N_k are the applied torques, I_k are the principal moments of inertia and w_k are the components of the angular velocity vector along the principal axes.

Euler-Rodrigues parameters and formulas, also called Euler parameters

The Euler-Rodrigues parameters are 4 numbers a, b, c, d such that $a^2+b^2+c^2+d^2=1$.

The Euler-Rodrigues formulas express the elements of a 3D rotation matrix in terms of the Euler-Rodrigues parameters, and occur in practice in software for artificial satellite altitude control or 3D computer games.

Euler's forward integration method

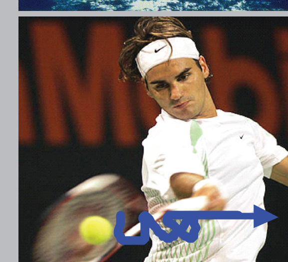
In mathematics and computational science, Euler integration method is the most basic kind of numerical integration for calculating trajectories from forces at discrete timesteps. More generally, the method is a numerical procedure for solving first-order differential equations with a given initial value. It uses the series $y_{n+1} = y_n + hf(x_n, y_n)$.

Three bodies

Euler, from 1760 onwards, was the first to study the general problem of three bodies under mutual gravitation (rather than looking at bodies in the solar system). When one body has negligible mass it is assumed that the motions of the other two can be solved as a two body problem, the body of negligible mass having no effect on the other two. Then the problem is to determine the motion of the third body attracted to the other two bodies which orbit each other. Even in this form the problem does not lead to exact solutions. Euler, however, found a particular solution with all three bodies in a straight line.

Euler's integration method is very useful to estimate the time position of a flying object, for example of a golf ball.

The rotation of a sattelite or of a tennis ball can be modeled with Euler's equations of rigid bodies.



The Euler-Rodrigues formulas express the elements of a 3D rotation matrix and are now used in many software to control satellite altitude, flight simulation or 3D video games.

Euler's disk

Euler's disk is a circular disk that spins, without slipping, on a surface. A typical example is a coin spinning on a table. Solving the coin's motion equations requires Euler's equations. The secret of the unusual properties of Euler's disk is its very low energy loss due to friction. Like a maglev train, a small amount of energy produces a surprising amount of motion. If laid end-to-end, 100,000 Euler's disks would stretch out for 4 miles and weigh 50 tons, the same as one typical battle tank!

Power used by one 100 watt light bulb = Power used by 100'000 Euler's disks spinning continuously

Using Euler's equations of motion, one can show that as the disk loses energy, the soaring pitch produced by the rolling point of contact increases towards infinity. This is a beautiful example of the subtle and elegant motion of the toy.

www.eulersdisk.com

Many other interesting and solved problems about the Euler's disk motion can be found on www.eulersdisk.com of this little toy, including one very recently published in "Nature".

$$e^{i\pi} + 1 = 0$$

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