## Euler, the grandfather of sudoku

Leonhard Euler is sometimes considered as the father of the popular sudoku, because he studied and solved for the first time in history similar puzzles, namely the Latin Square and the Graeco-Latin square, also called Euler's squares.

## From Arabic numerologists to Euler

A so-called Latin square is an $n \times n$ table filled with $n$ different symbols in such a way that each symbol occurs exactly once in each row and in each column. Here are two examples (one using figures, one using letters):
$\left[\begin{array}{lll}1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2\end{array}\right] \quad\left[\begin{array}{llll}a & b & d & c \\ b & c & a & d \\ c & d & b & a \\ d & a & c & b\end{array}\right]$

The first Latin squares were engraved in ancient architecture as numerologica talismans. Arabic numerologists had already compiled a long list of order 3 through order 9 Graeco-Latin squares by A.D. 990

The Latin squares and Graeco-Latin squares apparently became popular puzzles after Euler studied them.

## Euler square

An Euler square or Graeco-Latin square of order $n$ over two sets S and T , each consisting of $n$ symbols, is an $n \times n$ arrangement of cells, each cell containing an ordered pair ( $\mathrm{s}, \mathrm{t}$ ), where $\mathrm{s} \in \mathrm{S}$ and $\mathrm{t} \in \mathrm{T}$, such that

- every row and every column contains exactly one $s \in S$ and exactly one $t \in T$, and
- no two cells contain the same ordered pair of symbols.

The two sets are commonly taken to be $S=\{A, B, C, \ldots\}$ and $T=\{\alpha, \beta, \gamma, \ldots\}$ hence the name Graeco-Latin square, originated from Euler

| $A \alpha$ | $B \gamma$ | $C \beta$ |
| :--- | :--- | :--- | :--- |
| $B \beta$ | $C \alpha$ | $A \gamma$ |
| $C \gamma$ | $A \beta$ | $B \alpha$ |

In the 1780s, Euler demonstrated methods for constructing such squares where n is odd or a multiple of 4 .


| Y | 5 | 5 | ? | ? | ? |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 5 | \% | ? | ? | ? |
| 5 | \% | 8 | ? | ? | ? |
| ? | ? | ? | ? | ? | ? |
| ? | ? | ? | ? | ? | ? |
| ? | ? | ? | ? | ? | ? |

The thirty-six officers problem is a mathematical puzzle proposed and solved by Euler in 1779.

The problem asks whether it is possible to arrange 6 regiments, each consisting of 6 officers of different ranks, in a $6 \times 6$ square so that no rank or regiment will be repeated in any row or column Such an arrangement would form an Euler or Graeco-Latin square Euler correctly predicted there was a solution for all nxn puzzles except for the $2 \times 2$ and the $6 \times 6$ ("the thrity-six officers") problems Gaston Tarry compled the proofs in 1901.


Euler or Graeco-Latin square

## From Euler to sudoku

During his last year, Euler showed how to constuct Magic Squares with a certain number of cells, in particular $9,16,25$, and 36 . His method starts with Graeco-Latin Squares and puts constraints so that the result is a Latin square

The aim of the sudoku puzzle is to enter a numerical digit from 1 through 9 in each cell of a $9 \times 9$ grid made up of $3 \times 3$ subgrids (called regions"), starting with various digits given in some cells (the "givens"); each row, column and region must contain only one instance of each numeral.

Sudoku puzzles constitute a type of Latin square, but with additional constraints additional constraints was apparently borne in France in 1895. The sudoku was invented in usurged from a revival in Japan in 1986

Examples of standard and variated sudoku puzzles

## MaaTec <br> www.maatec.com



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| 5 | 3 | 7 |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 |  |  | 1 | 9 | 5 |  |  |  |
|  | 9 | 8 |  |  |  |  | 6 |  |
| 8 |  |  |  | 6 |  |  |  | 3 |
|  |  |  | 8 |  | 3 |  |  | 1 |
| 7 |  |  |  | 2 |  |  |  | 6 |
|  | 6 |  |  |  |  | 2 | 8 |  |
|  |  |  | 4 | 1 | 9 |  |  | 5 |
|  |  |  |  | 8 |  |  | 7 | 9 |

The top right region must contain a 5 . By hatching across elsehwere, the solver can eliminate all the empty cells in the region which cannot contain a 5 . The leaves only one (shaded).

