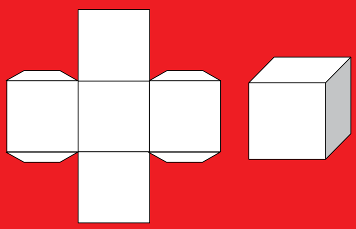
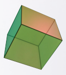


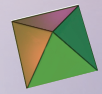
Euler's geometry

Euler's Characteristic

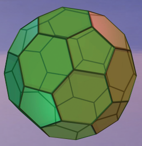
Is there a common feature between all these objects, and many others? Euler found the answer:
V - E + F = 2, called Euler's formula where V = number of vertices, E = number of edges and F = number of faces.



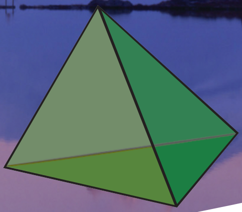
Cube
V = 8 E = 12 F = 6
8 - 12 + 6 = 2



Pyramid
(half of octahedron)
V = 5 E = 8 F = 5
5 - 8 + 5 = 2

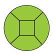


Football
V = 60 E = 90
F = 32 (12 pentagons + 20 hexagons)
60 - 90 + 32 = 2



Tetrahedron
V = 4 E = 6 F = 4
4 - 6 + 4 = 2

Suppose any polyhedron is a rubber balloon. You can then flatten it by getting rid of one face and enlarging it around all others. For example, the cube becomes:



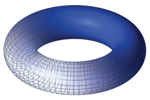
You now need to prove that in the new flattened ballon V - E + F = 1, since you got rid of one face, but of no vertex and of no edge.

Clearly, for any polygon in the plane, with n vertices, V = n, E = n, F = 1, and V-E+F = n-n+1 = 1.

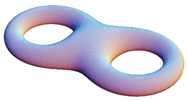
If one adds a polygon of x vertices and sharing y with another polygon, or with a group of polygons where V1 - E1 + F1 = 1, V = V1 + V2 + x - y, E = E1 + E2 + x - y + 1 and F = F1+1, therefore V - E + F = (V1 + V2 + x - y) - (E1 + E2 + x - y + 1) + (F1 + 1) = (V1 - E1 + F1) + (V2 - E2 + 1) -1 = 1 + 1 - 1 = 1.

Thus V - E + F = 2 for any polyhedron (genus-zero).

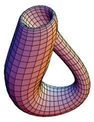
The general Euler's Characteristic χ = V - E + F applies to non-convex polyhedra and even to ther shapes:



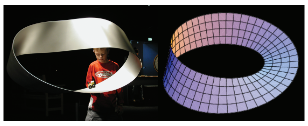
Torus: χ = 0



Double Torus: χ = -2



Klein bottle: χ = 0



Möbius Strip in Technorama, the Swiss Science Center: χ = 0

Edelweiss (stellated dodecahedron): χ = 4

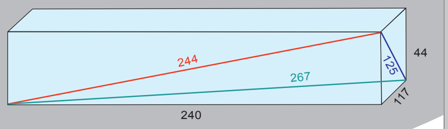
Euler brick

An Euler brick is a cuboid with integer edges and integer face diagonals. A primitive Euler brick is an Euler brick with its edges relatively prime or equivalently a solution to the following system of diophantine equations

$a^2 + b^2 = d^2$
$b^2 + c^2 = e^2$
$a^2 + c^2 = f^2$

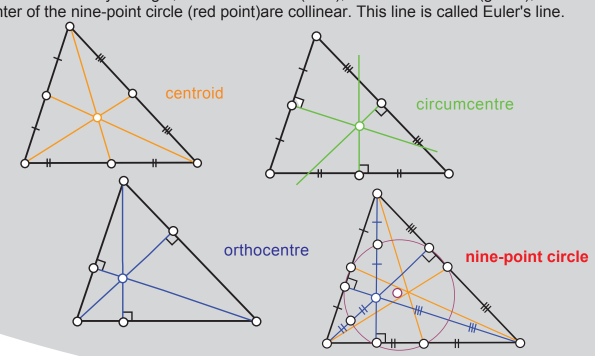
Euler found at least two parametric solutions to the problem. The smallest Euler brick has edges (a,b,c) = (240,117,44) and faces diagonals 267, 244, and 125. Paul Halcke discovered it in 1719.

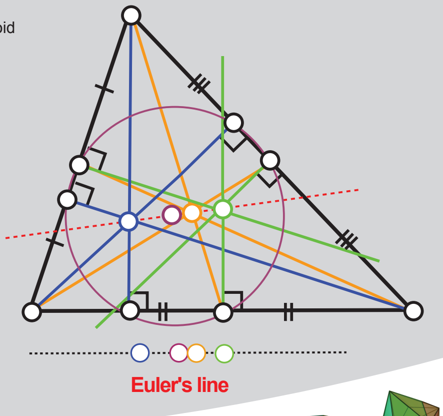
Other solutions are (275, 252, 240), (693, 480, 140), (720, 132, 85), (792, 231, 160).



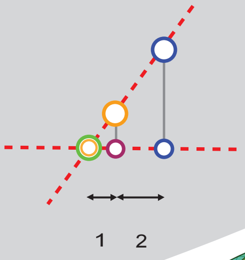
Euler's line

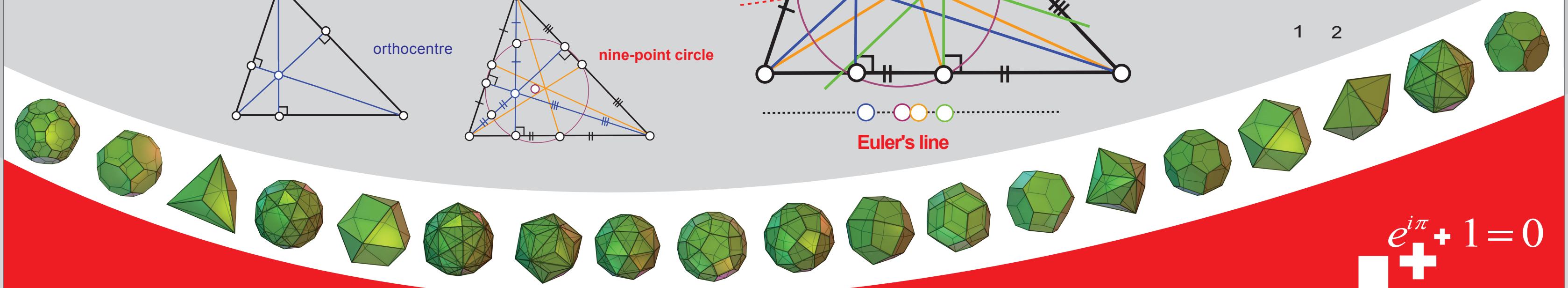
Leonhard Euler showed that in any triangle, the orthocenter (blue), the circumcenter (green), the centroid (yellow), and the center of the nine-point circle (red point)are collinear. This line is called Euler's line.





Finally he proved that the following pairs of segments are proportional, and that each of these pairs have thvemselves a 1:2 length ratio:





$e^{i\pi} + 1 = 0$

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

State Secretariat for Education and Research

Switzerland.