Euler's number

$e = 2{,}7182818284590452...$

The mathematical constant e is the base of the natural logarithm (ln(x) or $\log_e(x)$).

It is often called Euler's number, due to the related and extensive discoveries of Euler. The exact reasons why Euler himself started to use the letter e for the constant are unknown, but it may be because it is the first letter of the word "exponential" rather than of his name, because he was a very modest man.

History

Jacob Bernoulli, another famous Basel mathematician, had defined the number e as $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ that he estimated between 2 and 3. He was conscious of the unproved equivalence with the 2 other formulas:

The sum of the infinite series where n! is the factorial of n:	The unique real number e > 0 such that (that is, such that area under the hyperbola f(t) = 1 / t from 1 to e is equal to 1):
$e=\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$	$\int_1^e \frac{1}{t}\,dt = 1$

Mathematical properties

The exponential function $f(x) = e^x$ is important in part because it is the unique nontrivial function (up to multiplication by a constant) which is its own derivative, and therefore, its own primitive:

$$\frac{d}{dx}e^x = e^x \quad \text{and} \quad \int e^x\,dx = e^x + C$$

where C is the arbitrary constant.

Euler's formula of complex analysis

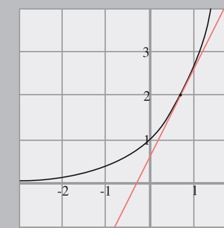
The complex analysis equation $e^{ix} = \cos x + i\sin x$ is called Euler's identity. The special case with x = π is the famous Euler's formula: $e^{i\pi} + 1 = 0$

This formula of complex exponential functions was described by Richard Feynman as "[...] the most remarkable formula in mathematics [...], our jewel", because e is one of the most important numbers in mathematics, alongside coincidentally with 0, 1, i, and π.

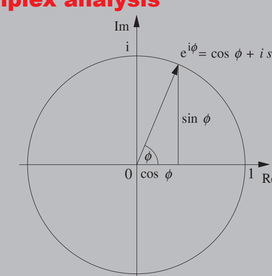
Other Euler's numbers:

Euler number in physics (fluids):	Euler primes:	Euler numbers in number theory:	Euler-Mascheroni constant:
$Ca=\frac{p-p_v}{\frac{1}{2}\rho V^2}$ where ρ is the density of the fluid, p is the local pressure, pv is the vapor pressure of the fluid and V is a characteristic velocity of the flow.	Euler primes or symmetric primes are primes that are the same distance from a given integer.	E0 = 1, E2 = -1, E4 = 5, E6 = -61, ...	γ = 0.57721 56649 01532...

The function f(x) = e^x and its derivative



Complex analysis

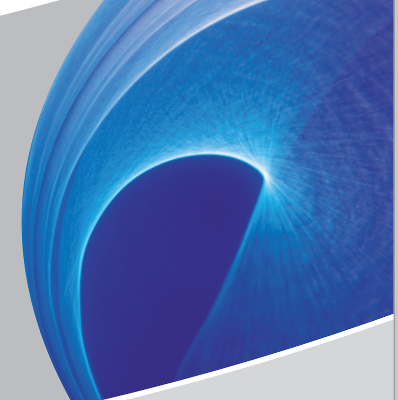


Euler made very important contributions to complex analysis, with the e number. He discovered what is now known as Euler's formula, i.e. that for any real number x, the complex exponential function satisfies

$$e^{ix} = \cos x + i\sin x$$

$2,718,281,828

Google in 2004 announced its intention to raise $2,718,281,828 stock.



e in physics and engineering

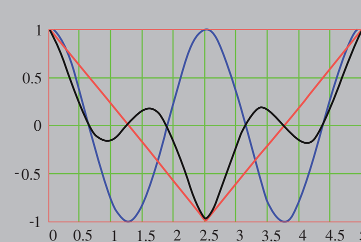
Complex numbers are used in almost every field of physics and engineering.

Euler's formula is widely used in quantum mechanics, relativity.

In electrical engineering, signals that vary periodically over time are often described as a combination of sine and cosine functions, and are more conveniently expressed as the real part of exponential functions with imaginary exponents, using Euler's formula. The Fourier series are based on it.

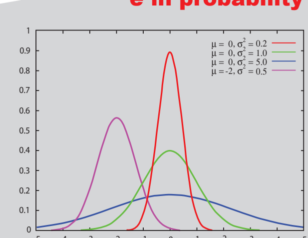
In fluid dynamics, Euler's formula is used to describe potential flow. Concrete examples of problems include the calculation of optimal shapes of transport vehicles, of energy generators, of boat turbines, the simulation of chemical/physical processes, the weather forecast, etc.

In differential equations, the function e^{ix} is often used to simplify derivations, even if the final answer is a real function involving sine and cosine. Euler's identity is an easy consequence of Euler's formula.



Fourier series

e in probability and statistics



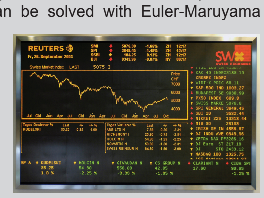
The normal distribution, describing measurement errors and that is used daily in statistics, was developed based on certain discoveries of Euler.

e in finance

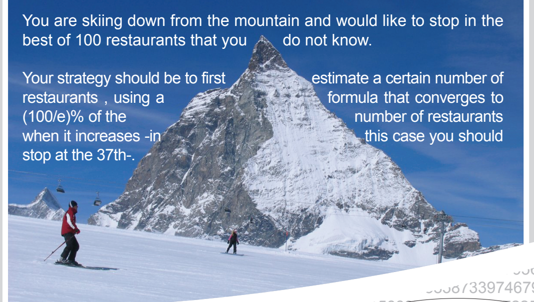
e is also the amount of money if you deposit 1 Swiss Frank at a continuously compounded interest rate of 100%: the terminal value of an interest compounded m times per annum is $(1 + 100\%/m)^m$, and it tends to e=2.71828... if $m \to \infty$.
Countinuously compounded interest is systematically used for the valuation of futures and options. The Black-Scholes differential equation and its solution depend on Euler's number e and Euler's formula and can be solved with Euler-Maruyama technique.

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0$$

Black-Scholes differential equation

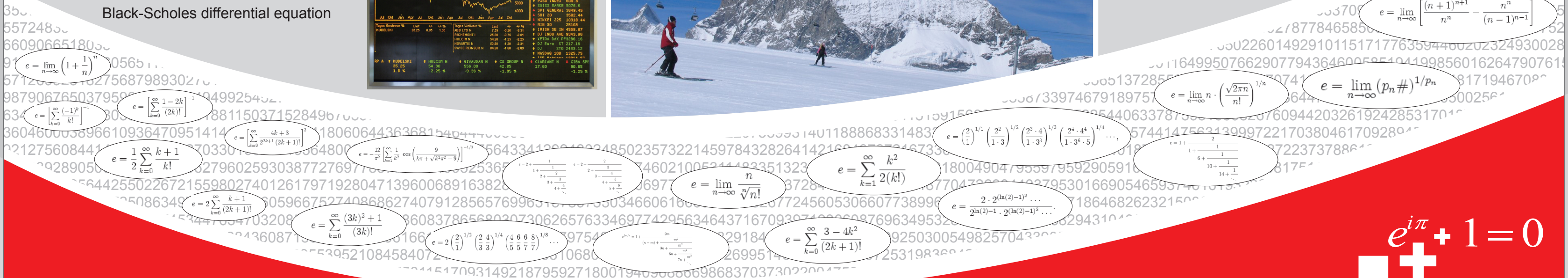


e in operations research: the "optimal stopping problem"



You are skiing down from the mountain and would like to stop in the best of 100 restaurants that you do not know.

Your strategy should be to first estimate a certain number of restaurants , using a formula that converges to (100/e)% of the number of restaurants when it increases -in this case you should stop at the 37th-.



$e^{i\pi} + 1 = 0$

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