Euler, a father of numerical analysis

Numerical analysis is the study of approximate numeral solutions of problems of continuous mathematics.

Euler contributed strongly to numerical analysis, and in particular to estimate solutions of differential equations. A differential equation is an equation in which the derivatives of a function appear as variables.

Euler integration

Euler integration is the most basic kind of numerical integration for calculating trajectories from forces at discrete timesteps. More generally, the method is a numerical procedure for solving first-order differential equations with a given initial value.

Euler integration is simply derived from equations for the derivatives of the position x(t) and velocity v(t) of an object.

The velocity $v(t) = \frac{dx(t)}{dt}$ is estimated as $v(t_0 + h) = v(t_0) + ha(t_0)$ where $a(t_0)$ is the original acceleration and the position is estimated as $x(t_0 + h) = x(t_0) + hv(t_0)$.

Euler-Maclaurin formula and Euler-Maruyanan method

The Euler-Maclaurin formula, discovered independently by Euler and Maclaurin, provides a powerful connection between integrals and sums. It can be used to approximate integrals by finite sums, or conversely to evaluate finite sums and infinite series using integrals and the machinery of calculus.

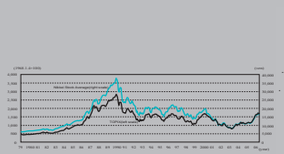
The Euler-Maruyama method is a technique for the approximate numerical solution of a stochastic differential equation. It is a simple generalization of the Euler method for ordinary differential equations to stochastic differential equations.

Today, numerical analysis methods are used in various fields, such as mechanics, chemistry, materials, biomolecules, biology, wheather forecast, economics, finance, etc.

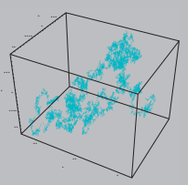
The use of Euler-Maruyanan in probability theory and financial mathematics

An important example is the equation for geometric Brownian motion. Brownian motion is a random movement, e.g. of particles suspended in a fluid. The Black and Scholes pricing model of call or put options is based on the model that stock prices follow Brownian motion.

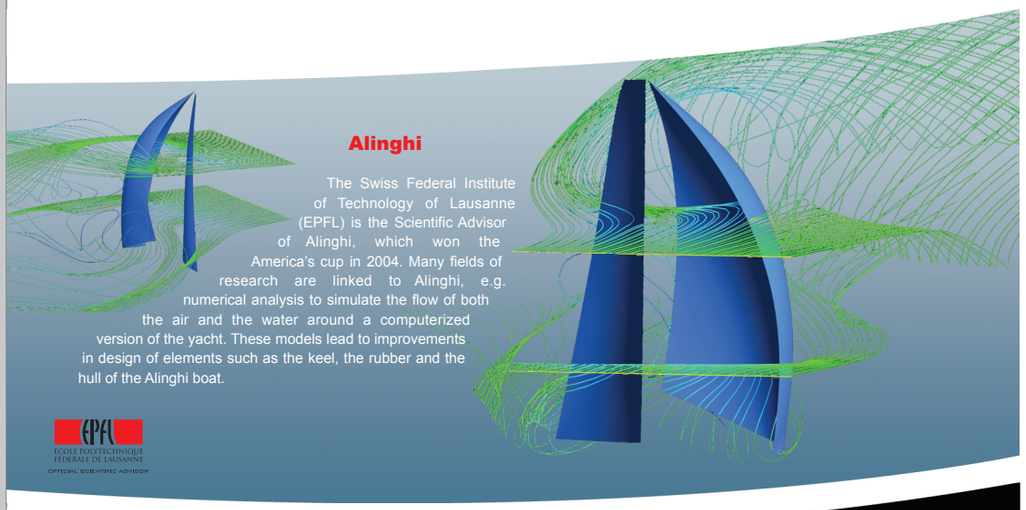
The Black and Scholes equation $dX_t = \mu X_t dt + \sigma X_t dB_t$ for the dynamics of call and put options prices is therefore a stochastic differential equation and the Euler-Maruyanan method can be applied.



1D Brownian motion: the stock market price.



3D Brownian motion models pollen particles or a fly



Alinghi

The Swiss Federal Institute of Technology of Lausanne (EPFL) is the Scientific Advisor of Alinghi, which won the America's cup in 2004. Many fields of research are linked to Alinghi, e.g. numerical analysis to simulate the flow of both the air and the water around a computerized version of the yacht. These models lead to improvements in design of elements such as the keel, the rubber and the hull of the Alinghi boat.

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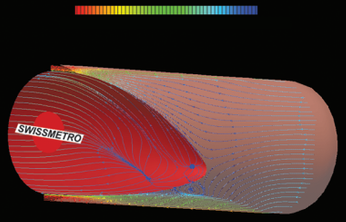
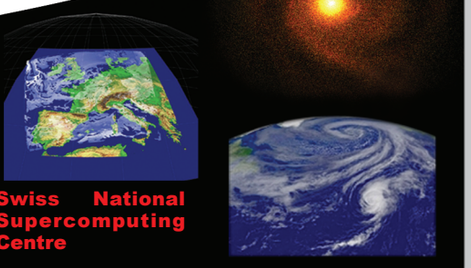
Euler integration

The Euler equation can be used to estimate the trajectory of an orbit, a planet, an asteroid, a comet, or ... a ski jumper.



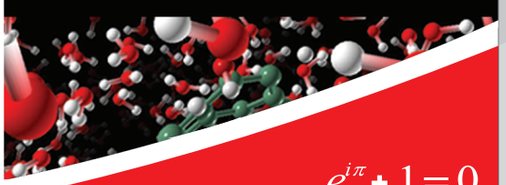
Swissmetro project

Swissmetro is an innovative high-speed metro in a low-pressure tunnel. Thanks to state-of-the-art technology, it is an entirely subterranean solution with an attractive cost model, efficiantly linking cities. The numerical simulations of the Swissmetro already confirmed its speed could reach 600 km/h.



Swiss National Supercomputing Centre

The Swiss National Supercomputing Centre is implied in many numerical analysis projects, e.g. to estimate supersonic turbulence, magnetic jet from a young stellar object, water flow in a turbine, the formation and evolution of galaxies, bio-inspired flow optimization, etc.



$$e^{i\pi} + 1 = 0$$

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